

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

Exercise 1.1

- 1 Find the values of the letters p , q and r that make the following pairs of expressions always equal.

$$(a) \quad \frac{1}{7}x = \frac{x}{p} \quad (b) \quad \frac{1}{5}(2x+3) = \frac{(2x+3)}{q} \quad (c) \quad \frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$$

- 2 Solve the following equations.

$$(a) \quad \frac{60}{x+4} = 12 \quad (b) \quad \frac{35}{2x-3} = 5 \quad (c) \quad \frac{20}{6-x} = \frac{1}{2}$$

- 3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab \cos C$.

- 4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x+2) \div 3$ by 12.
 (c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

- 5 Solve the following equations.

$$(a) \quad \frac{3}{4}(2x+3) = \frac{5}{8}(x-2) \quad (b) \quad \frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$$

$$(c) \quad \frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$$

- 6 Make x the subject of the following equations.

$$(a) \quad \frac{a}{b}(cx+d) = x+2 \quad (b) \quad \frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$$

- 7 Simplify the following as far as possible.

$$(a) \quad \frac{a+a+a+a+a}{5} \quad (b) \quad \frac{b+b+b+b}{b}$$

$$(c) \quad \frac{c \times c \times c \times c \times c}{c} \quad (d) \quad \frac{d \times d \times d \times d}{4}$$

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Exercise 1.2

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7} \times 5$	(b) $\frac{5}{12} \times 3$	(c) $\frac{7}{9} \times 2$	(d) $\frac{4}{15} \times 3$
(e) $\frac{8}{11} \div 4$	(f) $\frac{8}{11} \div 3$	(g) $\frac{6}{7} \div 3$	(h) $\frac{6}{7} \div 5$
(i) $\frac{3x}{y} \times x$	(j) $\frac{3x}{y^2} \times y$	(k) $\frac{5x^3}{4y} \div x$	(l) $\frac{5x^2}{6y} \div y$
(m) $\frac{5x^3}{2y} \times 3x$	(n) $\frac{3y^4}{4x^2z} \times 2x$	(o) $\frac{6x^2y^3}{5z} \div 2xy$	(p) $\frac{5a^2}{6x^3z^2} \div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$	(b) $V = \frac{4}{3}\pi x^3$	(c) $\frac{1}{2}(u + v) = tx$	(d) $W = \frac{2}{3}\pi x^2h$
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3 Simplify the following compound fractions.

(a) $\frac{\frac{1}{x} + 1}{\frac{1}{x} + 3}$	(b) $\frac{\frac{2}{x} + 1}{\frac{3}{x} - 1}$	(c) $\frac{\frac{1}{x+1} + 2}{\frac{1}{x+1} + 1}$
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4 Write as single fractions.

(a) $\frac{2}{x-1} + \frac{1}{x+3}$	(b) $\frac{2}{x-3} - \frac{1}{x+2}$	(c) $\frac{1}{2x-1} - \frac{1}{3x+2}$	(d) $\frac{3}{x+2} + 1$
(e) $2 - \frac{1}{x-1}$	(f) $\frac{2x}{x+1} - 3$	(g) $\frac{3}{4(2x-1)} - \frac{1}{4x^2-1}$	

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}} + \sqrt{x}$	(b) $\frac{2x}{\sqrt{x+3}} + \sqrt{x+3}$	(c) $\frac{x}{\sqrt[3]{x-2}} + \sqrt[3]{(x-2)^2}$
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6 Write the following in the form $1 + \frac{a}{x+b}$.

(a) $\frac{x+1}{x-5}$	(b) $\frac{x+3}{x+1}$	(c) $\frac{x+2}{x+5}$	(d) $\frac{x-6}{x-2}$
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- 7 Write the following equations without fractions. (A , B etc. are constants that remain in your answers.)

$$(a) \quad \frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$(b) \quad \frac{x+2}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$(c) \quad \frac{2}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$(d) \quad \frac{1}{(x-2)^2(x+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+1}$$

$$(e) \quad \frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

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1 Write without brackets.

(a) $(x + 5)^2$ (b) $(x - 4)^2$ (c) $(2x + 1)^2$

(d) $(3x - 2)^2$ (e) $(x + 2)(x - 2)$ (f) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(x + 5)^2 + (y + 2)^2 = (x - 5)^2 + (y - 2)^2$

(c) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2 + 4}$ (b) $\sqrt{x^2 - 4x + 4}$ (c) $\sqrt{x^2 - 1}$

(d) $\sqrt{x^2 + 9x}$ (e) $\sqrt{x^2 - y^2}$ (f) $\sqrt{x^2 + 2xy + y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

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6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$

(b) $9x^2 - 12x - 1$

(c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$

(b) $4x^2 - 36$

(c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$

(e) $3x^2 - 5x + 2$

(f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x + \frac{1}{x}\right)^2$

(b) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(c) $\left(x + \frac{2}{x}\right)\left(x - \frac{3}{x}\right)$

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Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\frac{4+6x}{2}$ (d) $\frac{4 \times 6x}{2}$ (e) $\frac{3x+xy}{x}$

(f) $\frac{3x \times xy}{x}$ (g) $\frac{4x+10y}{8x+6y}$ (h) $\frac{3x-6y}{9x-3y}$

(i) $\frac{4x+9y}{2x+3y}$ (j) $\frac{4x+6y}{6x+9y}$ (k) $\frac{5xy+6y^2}{10x+12y}$

(l) $\frac{3x^2+4y^2}{6x^2-8y^2}$ (m) $\frac{x-3}{3-x}$ (n) $\frac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\frac{ax}{b} = \frac{py}{qz}$ (b) $\frac{3\pi ax}{b} = \frac{4y^2}{qz}$

3 Simplify the following.

(a) $\frac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\frac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c) $\frac{1}{2}k(k+1) + (k+1)$ (d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\frac{x^2+6x+8}{x^2-x-6}$ (b) $\frac{3x^2-2x-8}{x^2-4}$

(c) $\frac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\frac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

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$$(e) \frac{\frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2}$$

$$(f) -\frac{\frac{x}{2\sqrt{1-x}} + \sqrt{1-x}}{x^2}$$

$$(g) \frac{\frac{\sqrt{x}}{2\sqrt{1+x}} - \frac{\sqrt{1+x}}{2\sqrt{x}}}{x}$$

$$(h) \frac{\sqrt[3]{1+x} - \frac{x}{3\sqrt[3]{(1+x)^2}}}{\sqrt[3]{1+x}}$$

**MATHEMATICS A
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Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$3x + y = 10$

2 $x^2 - 4x + y^2 = 21$

$y = 3x - 21$

3 $x^2 + xy + y^2 = 1$

$x + 2y = -1$

4 $x^2 - 2xy + y^2 = 1$

$y = 2x$

5 $c^2 + d^2 = 5$

$3c + 4d = 2$

6 $x + 2y = 15$

$xy = 28$

7 $2x^2 + 3xy + y^2 = 6$

$3x + 4y = 1$

8 $2x^2 + 4xy + 6y^2 = 4$

$2x + 3y = 1$

9 $4x^2 + y^2 = 17$

$2x + y = 5$

10 $2x^2 - 3xy + y^2 = 0$

$x + y = 9$

11 $x^2 + 3xy + 5y^2 = 15$

$x - y = 1$

12 $xy + x^2 + y^2 = 7$

$x - 3y = 5$

13 $x^2 + 3xy + 5y^2 = 5$

$x - 2y = 1$

14 $4x^2 - 4xy - 3y^2 = 20$

$2x - 3y = 10$

15 $x^2 - y^2 = 11$

$x - y = 11$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$x + y = 7$

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Exercise 1.6

1 Write the following as powers of x .

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n .

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x .

(a) $x^3\left(x + \frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x + \frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

7 Simplify $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}}$.

**MATHEMATICS A
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1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

2 Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

3 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin 2x^\circ = 0.829$ (b) $\cos 3x^\circ = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x^\circ = 0.8$ (e) $\cos \frac{1}{2}x^\circ = 0.3$ (f) $\tan \frac{1}{3}x^\circ = 0.7$

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Exercise 2.2

Do not use a calculator in this exercise.

1 In this question θ is in the range $0 \leq \theta < 90$.

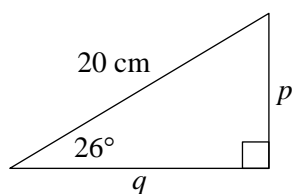
(a) Given that $\sin \theta = \frac{12}{13}$, find the exact values of $\cos \theta$ and $\tan \theta$.

(b) Given that $\tan \theta = \frac{6}{7}$, find the exact values of $\sin \theta$ and $\cos \theta$.

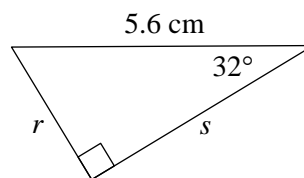
(c) Given that $\cos \theta = \frac{5}{8}$, find the exact values of $\sin \theta$ and $\tan \theta$.

2 Find expressions, of the form $a \sin \theta$ or $b \cos \theta$, for the sides labelled with letters in these triangles.

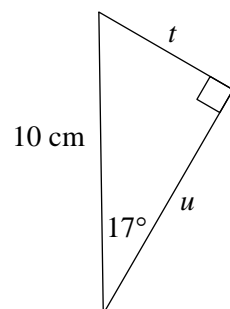
(a)



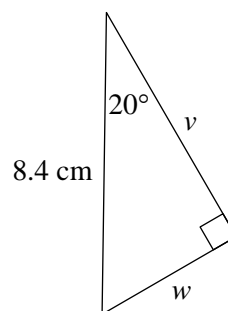
(b)



(c)



(d)



Exercise 3.2

Sketch (do not *plot*) the general shape of the graphs of the following curves.

Axes are not required but can be included in the questions marked with an asterix.

1 $y = x^2 - 3x + 2$

2 $y = -x^2 + 5x + 1$

3 $y = 1 - x^2$

4 $y = (x - 2)(x + 4)$

5 $y = (3 - x)(2 + x)$

6 $y = (1 - x)(5 - x)$

7 $y = x^3$

8 $y = -x^3$

9* $y = \frac{3}{x}$

10* $y = -\frac{2}{x}$

11 $y = (x - 2)(x - 3)(x + 1)$

12* $y = \frac{2}{x^2}$

13 Sketch on the same axes the general shape of the graphs of $y = x^2$ and $y = x^4$.

14 Sketch on the same axes the general shape of the graphs of $y = x^3$ and $y = x^5$.

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- 25 (a) Sketch the graph of $y = \frac{1}{x^2}$.
- (b) Sketch $y = \frac{2}{x^2}$ on the same axes.
- 26 (a) Sketch the graph of $y = x^3$.
- (b) Sketch $y = 2x^3$ on the same axes.
- 27 (a) Sketch the graph of $y = x^4$.
- (b) Sketch $y = 3x^4$ on the same axes.
- 28 (a) Sketch the graph of $y = x^3 - 4x$.
[Hint: It cuts the x -axis at -2 , 0 and 2 .]
- (b) Sketch $y = 2x^3 - 8x$ on the same axes.
- 29 (a) Sketch the graph of $y = x^4 - x^2$.
[Hint: It cuts the x -axis at 1 and -1 , and touches the axis at 0 .]
- (b) Sketch $y = -x^4 + x^2$ on the same axes.
- 30 Sketch, on separate axes, the following graphs. Show the x -coordinates of the intersections with the x -axis.
- (a) $y = 4 - x^2$
- (b) $y = (x - 2)(x + 1)$
- (c) $y = -(x - 2)(x + 1)$
- (d) $y = x(x + 4)$
- (e) $y = (x - 2)^2$
- (f) $y = -(x + 1)^2$
- (g) $y = (1 - x)(2 + x)$